# Worksheet for 2021-11-01

## Conceptual questions

**Question 1.** For each line integral below, would you proceed by direct parametrization, FTLI, or Green's Theorem? (There may not be merely one "correct" answer.)

(a) $C$ is $x^2 + y^2 = 9, x \geq 0, y \geq 0$ from $(3, 0)$ to $(0, 3)$, and the integral is $\int_C \langle 0, x - 2\rangle \cdot d\mathbf{r}$.
(b) Same $C$ as previous part, with $\int_C \langle x^2, y^3\rangle \cdot d\mathbf{r}$.
(c) $D$ is the path from $(-1, 1)$ to $(2, 4)$ in a straight line, and then back to $(-1, 1)$ via the parabola $y = x^2$, and the integral is $\int_D x\, ds$.
(d) Same $D$ as previous part, with $\int_D \langle \sin(x^3), xy^2\rangle \cdot d\mathbf{r}$.
(e) $E$ is the piecewise linear path from $(-2, 0)$ to $(-1, 1)$ to $(1, 1)$ to $(2, 0)$, and the integral is $\int_E \langle x^2, y^3\rangle \cdot d\mathbf{r}$.
(f) Same $E$ as previous part, with $\int_E \langle \sin(x^3), xy^2\rangle \cdot d\mathbf{r}$.

**Question 2.** Let $D$ be the annulus

$$1 \leq x^2 + y^2 \leq 4.$$

What is the "positively oriented boundary" of $D$? **Hint:** If it helps, try breaking $D$ into an upper half and a lower half and considering the two halves separately.

## Computations

**Problem 1.** Consider a polygon in the plane, whose vertices read counterclockwise are

$$(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$$

(so $n$ is the number of vertices, e.g. $n = 5$ for a pentagon).

(a) Use the fact that $\langle P, Q\rangle = \langle -y/2, x/2\rangle$ has $Q_x - P_y = 1$ to come up with a formula for the area of the polygon in terms of the constants $x_1, \ldots, x_n, y_1, \ldots, y_n$.
(b) Let $\mathbf{v}_i = \langle x_i, y_i, 0\rangle$. Show that your expression from the preceding part is the same as the $z$-component of the vector

$$\frac{1}{2}(\mathbf{v}_1 \times \mathbf{v}_2) + \frac{1}{2}(\mathbf{v}_2 \times \mathbf{v}_3) + \cdots + \frac{1}{2}(\mathbf{v}_{n-1} \times \mathbf{v}_n) + \frac{1}{2}(\mathbf{v}_n \times \mathbf{v}_1).$$

Can you interpret this area formula geometrically?
(c) In part (a) we could've taken $\langle P, Q\rangle = \langle -y, 0\rangle$ or $\langle 0, x\rangle$ instead. How would the geometric interpretations of the final expressions differ? (Of course, the whole expression is equal to the area of the polygon, but what about term-by-term?)

If doing it in full generality is too much to start with, pick a specific triangle and do it with that.